
Research article

A novel spectral framework for stochastic differential equations: leveraging shifted Vieta-Fibonacci polynomials

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Abstract: In this paper, we introduced a novel numerical approach for solving stochastic heat equations and multi-dimensional stochastic Poisson equations using shifted Vieta-Fibonacci polynomials (SVFPs), marking their first application in stochastic differential equations. The proposed method leveraged the orthogonality and recurrence properties of SVFPs to approximate solutions with high precision. By normalizing the polynomial basis and their derivatives, the technique ensured numerical stability and convergence, addressing challenges encountered in earlier implementations. The method was rigorously validated through comparisons with the fast discrete Fourier transform approach, other methods in the literature, and, where applicable, exact solutions, demonstrating superior accuracy. Five illustrative problems were analyzed, with results showcasing significantly reduced variance and absolute errors, particularly for higher-order approximations. The numerical simulations, executed using *Mathematica 12*, highlighted the robustness of the SVFPs-based algorithm in handling stochastic variability. This work not only extended the applicability of SVFPs to stochastic domains but also provided a reliable framework for future research on fractional and nonlinear stochastic systems.

Keywords: Vieta-Fibonacci polynomials; orthogonal polynomials; heat equation; Poisson equation; stochastic differential equations; white noise

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